Section 4.5

Example of Integration by Substitution: When evaluating

$$\int (x^2 + 1)^4 (2x)\, dx,$$

notice that if you let $u = x^2 + 1$, then the derivative is $du = 2x\, dx$. With these substitutions, the integral becomes

$$\int u^4\, du,$$

which is easily evaluated using the power rule: $\int u^4\, du = \frac{1}{5}u^5 + C = \frac{1}{5}(x^2 + 1)^5 + C$. You can check that this is the correct antiderivative by differentiating it.

1) Find the following, and then check that your answer is correct by differentiating it.

   a) $\int (x^2 - 3)^3 (2x)\, dx$

   b) $\int 3 \cos 3x\, dx$

Another Example of Integration by Substitution: Consider $\int x(x^2 + 1)^2\, dx$: You are trying to find a way to replace $x\, dx$ with du, so if you let $u = x^2 + 1$, then $du = 2x\, dx$. If you divide both sides by 2, you get $\frac{1}{2} du = x\, dx$, so the integral becomes $\int u^2 \left(\frac{1}{2} du\right)$. You can move the constant to the front, so the integral is

$$\frac{1}{2} \int u^2\, du,$$

which is easily evaluated using the power rule.

2) Find the following, and then check that your answer is correct by differentiating it.

a) $\int x^2(x^3 + 4)\,dx$

b) $\int (x - 1)(x^2 - 2x)\,dx$

3) Find the following, and then check that your answer is correct by differentiating it.

a) $\int \sqrt{4x + 3}\,dx$

b) $\int x\sqrt{5x - 3}\,dx$

c) $\int \tan^2 2x \sec^2 2x\,dx$

4) Find the following definite integrals.

a) $\int_1^2 x(x^2-3)^2\,dx$

b) $\int_2^7 \frac{x}{\sqrt{x+2}}\,dx$

5) Evaluate

$$\int_{-1}^{1} \frac{x^3-x}{x^2-4}\,dx$$

Homework for 4.5: #7, 23, 41, 51, 59

Section 4.6

If f is continuous on $[a, b]$, then

a) **The Trapezoidal Rule** for approximating $\int_a^b f(x)\,dx$ is given by

$$\int_a^b f(x)\,dx \approx \frac{(b-a)}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)].$$

b) **The Midpoint Rule** for approximating $\int_a^b f(x)\,dx$ is given by

$$\int_a^b f(x)\,dx \approx \frac{(b-a)}{n}[f(x_1^*) + f(x_2^*) + \cdots + f(x_{n-1}^*) + f(x_n^*)],$$

where $x_i^* = \frac{x_i + x_{i-1}}{2}$.

c) **Simpson's Rule** for approximating $\int_a^b f(x)\,dx$ is given by

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)],$$

where n is an even number.

1) Approximate

$$\int_0^2 x^2\,dx$$

with $n = 4$, using

a) The Trapezoidal Rule

b) The Midpoint Rule

c) Simpson's Rule

Compare your answers from the three approximations to the exact value of the definite integral.

Homework for 4.6: #9, 11